Journal of Physical Education and Sport ® (JPES), Vol. 21 (issue 6), Art 457, pp. 3374 - 3378, December 2021
online ISSN: 2247 - 806X; p-ISSN: 2247 – 8051; ISSN - L = 2247 - 8051 © JPES

Original Article

Coaching for character development: insights from a simple mathematical model

THOMAS M. EDWARDS, COSIMO CHIERA
Natural Intelligence Pty. Ltd., 14 Jembecca Dr Berwick, 3806, Victoria, AUSTRALIA
(Authors also affiliated with Eastern College Australia)

Published online: December 30, 2021
(Accepted for publication December 15, 2021)
DOI:10.7752/jpes.2021.06457

Abstract:
Problem statement: Character development is important for achieving sporting success. However, it cannot be presumed that athletes will develop healthy attitudes, values or beliefs about themselves, their sport or their opponents. Unfortunately, character development has lagged behind fitness and skills training in having recognised protocols able to consistently deliver positive outcomes for athletes. Approach: The logistic map is a simple mathematical model which has found applicability across disciplines when seeking to understand change. Indeed, the logistic map can model a variety of complex behaviours and has thus found favour in the biological sciences, including psychology. A simple variation of the logistic map has been developed and tested herein as a way to begin to understand the complexities of character development amongst athletes. Purpose: This paper sets forth a set of conditions, testable predictions, and some warnings useful for physical education teachers, coaches and sport psychologists alike when seeking to construct effective character development protocols for athletes. Results: The mathematical model provided three outcomes of note. First, it was capable of demonstrating substantial sustained positive growth (i.e. ≤184%) over time. It was therefore fit-for-purpose. Second, character development required at least moderate levels of adversity to develop and occurred by small increments over time (i.e. ≤81 iterations). Character development failed when threat was low and/or the initial amount of virtue (i.e. character) displayed by athletes was either similar to, or greater than, the threat faced. Usefully, each of these outcomes is also consistent with real-world experience and thus again validates the model. The implications of these outcomes are various but include: (1) the importance of promoting competition amongst junior athletes; (2) using a variety of experiences to spur character development so as to attain significant improvements in a time-limited way; and (3) avoiding hubris in elite athletes. The third, and perhaps most surprising, finding was that motivational messaging intended to improve athletes' character may be counter-productive if not tempered. A ratio of approximately 2:1 between hope-based vs. temperance-based motivational messages is suggested. Conclusions: Character development is multi-facetted and therefore complex, but nevertheless essential to sporting success. The present model reduces character development to a limited set of variables. In doing so it provides: (1) guidance to physical education teachers and coaches for the development of efficient protocols intended to enhance athletes' character development; and (2) testable predictions useful to sport psychologists in refining such protocols and optimising athletic performance.
Key words: Character development; coaching; physical education; motivation; sport psychology; logistic map

Introduction

Sport has played an important role in public life since antiquity; not least because it mirrors virtue, and therefore excellence, to the wider society (Reid, 2011). Yet, while fitness and skills development have established training protocols the growing of athletes' character, being also indispensable to success, remains an art more than a science (Hardman, 2008; Jones, 2017). Not only is there a growing interest in character development within the context of sport (d'Elia 2019; Hardman & Jones, 2011; Hardman et al., 2010; Marini et al., 2021; Yeager et al., 2001), but this is paralleled by a new-found clinical emphasis on virtues such as hope (Edwards & Jovanovski, 2016; Gustafsson et al., 2013; Snyder et al., 2002). Given this, the work of coaches and physical education teachers as mentors to younger athletes can now be effectively enhanced by sport psychologists.

However, developing protocols for character development is not simple. Such protocols need to be both comprehensive and based on falsifiable predictions. To this end the current paper uses a simple mathematical model, based on the logistic map (May, 1976), to suggest key factors worthy of consideration. It also provides testable predictions. As such, the conclusions drawn from this paper form the basis of future applied work in character development.

Helpfully, the logistic map has been used successfully within a number of biological disciplines, including psychology, as a simple way to model complex phenomena (Bhattacharyya, 2017; Richardson et al.,

Corresponding Author: THOMAS M. EDWARDS, E-mail: DrEdwards@NaturalIntelligence.com.au

2014). In the current context the logistic map is particularly useful because it can produce a variety of mathematical outcomes that have psychological, and therefore behavioural, correlates. For example, and in general, the logistic map can model: (1) decay and growth which is akin to behaviours becoming less/more likely over time; (2) steady state behaviour and therefore an unchanged (e.g. reinforced) psychological state; (3) oscillations (e.g. bipolar disorder); and (4) chaotic behaviour which is reminiscent of mental health problems such as breakdown and burnout.

Given the logistic map's ability to mimic a variety of psychological states we posit that it also has application in sport psychology. More specifically, because the logistic map can model decay, growth, steady state, oscillating and chaotic outcomes it has within it the capacity to demonstrate a variety of behaviours pertinent to displays, or otherwise, of virtue and thus character. In fact, the logistic map also has the capacity to suggest why some current practices in character development lack effectiveness, or are even counter-productive.

Methods

The logistic map is described by the equation $x_{n+1} = rx_n(1 - x_n)$ which is iterative. Therefore change is observed by plotting successive outputs (i.e. x_{n+1}), with each output being the next input (i.e. x_n). As to the parts of the equation, x_n is a measure of the proportion of some factor of interest within a population (e.g. virtue within a sporting club, team or squad) and, as stated, represents the equation's input value. As x_n is a proportional value it is represented by a decimal between 0 and 1.0; where 0 implies the factor no longer exists within the population and 1.0 implies that it saturates the population. Next, r represents the replication rate of the factor of interest.

When r is coupled to x_n, giving rx_n, the resultant compound term drives the growth of the factor through the population but, if left unchecked, would permit growth *ad infinitum*. To restrain unchecked growth a second compound term is coupled to it (i.e. $(1 - x_n)$). This additional compound term imposes restraint and is indicative of environmental threat. It describes the difference between what is ideally possible (i.e. a value of 1.0) and current reality (i.e. x_n). Finally, and as suggested above, x_{n+1} represents the logistic map's output; this being the proportion of the factor in the population under the combined influence of growth and restraint.

Interestingly, this simple mathematical model demonstrates a range of possible outcomes. For example, when r is:

1. between 0 and 1 the factor of interest decays from the population;
2. between 1 and 3 the factor of interest will approach the value $1 - \frac{1}{r}$ leading to a steady-state value which may be above or below the initial, or seed, value for x_n (i.e. x_0);
3. between 3 and ≈3.6 the factor of interest will oscillate with increasing periodicy;
4. between ≈3.6 and 4.0+ chaotic behaviour will result.

Now modified to the needs of character development the logistic map can be given by the iterative formula $v_{n+1} = mv_n\left[\sqrt{(\tilde{p} - v_n)}\right]$. Specifically, v_n represents the current level of virtue in the population as a more formal understanding of character. While v_{n+1} represents the level of virtue in the population under the influence of growth and restraint as an indication of character development.

In the present sporting context the replication rate is best understood in terms of motivation (i.e. m). This having both intrinsic and extrinsic properties. Motivation is perhaps best linked to character development through the virtue of hope. Hope is also a future-focused and goal-orientated motive force (Edwards & Jovanovski, 2016; Gustafson et al., 2013).

In addition, performance has been linked to both motivation and hope (Curry et al., 1997; Curry & Snyder; 2000; Norwood, 2001; Snyder et al., 2002). However, hope alone is not sufficient to provide performance outcomes consistent with an inverted U-shaped curve. For this reason motivation has been herein conceptualised according to two competing virtues: hope, H, and temperance, T. Such that $m = \frac{H}{T}$.

Restraint of *ad infinitum* growth is represented in the current model through the compound term $\left[\sqrt{(\tilde{p} - v_n)}\right]$. Here, $\tilde{p}$ takes the place of the value 1.0 in the original logistic map. It is scaled between 0 and 1.0 to account for changed environmental conditions, as opposed to only considering the ideal condition. The real-world significance of this adjustment is to acknowledge that athletes compete against a variety of opponents and therefore under different conditions. In addition, the variable p stands for "pressure" and is akin to an athlete's cognitive appraisal of threat. In the current model p is considered to be a function of both threat severity, s, and the time available to act to effectively to counter the perceived threat, a. Therefore $p = \frac{s}{a}$.

The relationship between p and $\tilde{p}$ is the scaling function $\tilde{p} = \frac{p}{1+p}$. Finally, by applying a square root to $(\tilde{p} - v_n)$ a particular deficit of the original logistic map, pertaining to modelling sustained positive behaviour (e.g. character growth), is overcome.

Results

Table 1 provides summary findings from nine simulations in which perceived threat (i.e. $\tilde{p}$) and virtue (i.e. in particular v_0 being the seed value) were manipulated. In addition, all nine simulations tested motivational values, m, in the range 0.9 to 5.0 at 0.1 intervals. For each interval of m, in each simulation, several hundred iterations of v_{n+1} were calculated.

Table 1: Nine simulations of character development.

Simulation	Outcome for v_{n+1}
1. Conditions of low perceived threat (e.g. squad/club training) $s = 0.3$, $a = 0.8$, therefore $\tilde{p} = 0.27$ Seed value for $v_n = 0.25$	No range of m identified where v_{n+1} reaches a steady state above v_0. However, v_{n+1} tended to 0 for values of $m \leq 1.9$; achieved a steady state below v_0 for m between 2.0 and 4.2; while v_{n+1} oscillated when $m > 4.2$.
2. Conditions of low perceived threat (e.g. squad/club training) $s = 0.3$, $a = 0.8$, therefore $\tilde{p} = 0.27$ Seed value for $v_n = 0.50$	Model collapsed given that $v_0 > \tilde{p}$.
3. Conditions of low perceived threat (e.g. squad/club training) $s = 0.3$, $a = 0.8$, therefore $\tilde{p} = 0.27$ Seed value for $v_n = 0.75$	Model collapsed given that $v_0 > \tilde{p}$.
4. Conditions of medium perceived threat (e.g. pre-season) $s = 0.6$, $a = 0.4$, therefore $\tilde{p} = 0.60$ Seed value for $v_n = 0.25$	For m = 1.7 to 2.8 v_{n+1} reached a steady state above v_0. Maximal steady-state for v_{n+1} was 0.47 when m = 2.8 representing an 88% increase on the seed value. Number of iterations to steady-state for v_{n+1} = 2.8 is 69.
5. Conditions of medium perceived threat (e.g. pre-season) $s = 0.6$, $a = 0.4$, therefore $\tilde{p} = 0.60$ Seed value for $v_n = 0.50$	A quick transition from a steady state value for v_{n+1} below v_0 to oscillations (i.e. m = 2.8 vs. 2.9).
6. Conditions of medium perceived threat (e.g. pre-season) $s = 0.6$, $a = 0.4$, therefore $\tilde{p} - 0.60$ Seed value for $v_n = 0.75$	Model collapsed given that $v_0 > \tilde{p}$.
7. Conditions of high perceived threat (e.g. competition) $s = 0.9$, $a = 0.1$, therefore $\tilde{p} = 0.90$ Seed value for $v_n = 0.25$	For m = 1.3 to 2.3 v_{n+1} reached a steady state above v_0. Maximal steady-state for v_{n+1} was 0.71 when m = 2.3 representing a 184% increase on the seed value. Number of iterations to steady-state for v_{n+1} = 2.3 is 77.
8. Conditions of high perceived threat (e.g. competition) $s = 0.9$, $a = 0.1$, therefore $\tilde{p} = 0.90$ Seed value for $v_n = 0.50$	For m = 1.6 to 2.3 v_{n+1} reached a steady state above v_0. Maximal steady-state for v_{n+1} was 0.71 when m = 2.3 representing a 42% increase on the seed value. Number of iterations to steady-state for v_{n+1} = 2.3 is 81.
9. Conditions of high perceived threat (e.g. competition) $s = 0.9$, $a = 0.1$, therefore $\tilde{p} = 0.90$ Seed value for $v_n = 0.75$	A quick transition from a steady state value for v_{n+1} below v_0 to oscillations (i.e. m = 2.3 vs. 2.4).

Note. In understanding perceived threat s pertains to severity, a to the time for effective action, and $\tilde{p}$ is the overall scaled measure of perceived threat. In understanding character v_n pertains to present virtue, v_0 is the seed value for v_n (i.e. at time = 0), and v_{n+1} pertains to the change in virtue (i.e. character) under the dual influences of growth and restraint. Finally, m is used to denote motivation.

Of the nine simulations tested only three resulted in sustained character development (see Table 1, simulations 4, 7 and 8). This is consistent with the notion that growing athletes in character is neither simple, nor obvious. Usefully, however, simulations 4, 7 and 8 demonstrated large and persistent gains in character above each seed value for v_n (i.e. gains between 42% to 184%). Interestingly, these three simulations were characterised by: (1) at least a moderate level of perceived threat (i.e. $\tilde{p}$); and (2) a seed value for v_n substantially less than the perceived threat. In addition, only moderate levels of motivation resulted in sustained character growth (i.e. values of m between 1.3 and 2.8). Finally, simulations 4, 7 and 8 demonstrated that between 69 and 81 iterations of the model were necessary to achieve maximal character development.

Discussion

The simple mathematical model developed and tested herein presents a rich set of findings for physical education teachers, coaches and sport psychologists interested in developing athletes' character. Three key findings, along with their implications, are summarised below.

To begin with, finding #1 is that the model can produce substantial and sustained character growth. Specifically, simulations 4, 7 and 8 demonstrated a growth in character between 42% and 184% (see Table 1). This outcome, in the first instance, represents a useful validation of the model.

Yet Table 1 also showed that simulations 4, 7 and 8 required between 69 and 81 iterations before achieving maximal character development. Further, simulation 7, which showed the greatest increase in character, demonstrated only an average 2.5% gain per iteration of the model. Several implications for physical education teachers and coaches now become apparent. For example, they must take a long-term view of character development aiming for small, but regular, improvements in their athletes. In saying this, however, readers should not assume that between 69 and 81 matches need be played, or competitions entered into, to achieve maximal and sustained character development. To do so would be to over-interpret the model and not be consistent with real-world experience. What must be recognised is that squad/team/club life offers many and regular opportunities to grow in character and these must be harnessed by physical education teachers and/or coaches acting as mentors.

Finding #2 is that character is formed under conditions of adversity. Lived experience would attest to this. For example, no simulation under conditions of low threat resulted in character development (see Table 1). Either character decayed/dipped/vacillated (i.e. simulation 1) or the model collapsed (i.e. simulations 2 & 3). Usefully, these low threat scenarios have real-world correlates which further up-holds the model. For example, the outcomes observed in simulation 1 mimic those forms of junior/school sport in which character is yet largely unformed and participation is favoured over competition. In accepting a *laissez faire* attitude amongst young athletes character development is compromised, even when motivational messaging from a physical education teacher or coach is high. By contrast, the collapse of simulations 2 and 3 is consistent with older athletes who have such "character" that their confidence now outweighs the threat faced and they risk falling into hubris (i.e. $v_0 > \tilde{p}$). This, most often, is a problem in elite-level sports.

More precisely, simulations 4, 7 and 8 (see Table 1) demonstrated that if athletes did not perceive at least a *moderate* level of threat from the outset then character development would be problematic. Even so, if v_0 approached $\tilde{p}$ in size then character development may oscillate at best. In the real world this outcome is akin to athletes not deeply engaging with notions of character development for they believe their current personal resources to be more-or-less adequate to match the threat they face. In such circumstances athletes may show sporadic signs of character. Indeed, character only grows significantly when v_0 is *much less* than $\tilde{p}$. That is, when the threat substantially out-weighs athletes' initial strength of character. Or, put another way, character development only occurs when athletes have a significant goal to strive for.

Taken together, we learn that if character development is to occur then athletes need to be challenged to change. However, a character challenge is not necessarily the same as goal-setting to achieve a desired physical outcome. For example, character development has multiple domains (Lickona, 2001), is developmentally-aware, as well as being a social process (Mascolo, 2014). It also has a virtuous intent. As such, character-related challenges should include social, behavioural, emotional and cognitive components as well as reflect virtuous ideals. Arguably, while coaches and/or physical education teachers are best placed to identify such challenges, sport psychologists also have an important role to play in making sure any presumptive character challenge is multi-facetted, conforms to theory and is not so great as to potentially cause harm.

Finally, let us consider the strength of motivational messaging by coaches and/or physical education teachers (i.e. being indicative of m) in generating improved character amongst athletes. Finding #3 has it that moderate levels of motivation lead to the best character outcomes. This is in keeping with performance being described according to an inverted U-shaped curve. Specifically, values of m in the range of 1.3 to 2.8, when compared to the entire motivational range (i.e. 0.9 to 5.0), were effective in developing character (see Table 1, simulations 4, 7 & 8). Indeed, higher values of m appeared to induce oscillations and would eventually result in "chaotic" mathematical behaviour akin to psychological breakdown and/or burnout (data not shown).

Given this, physical education teachers and coaches should moderate any effusive motivational messaging to athletes designed to build character. This implication is also consistent with the work of Kassing and Pappas (2007) and Cranmer et al. (2017). Put simply, infrequent, but well-targeted, "memorable messaging" by coaches is inherently motivating and virtues rich. It therefore serves the purpose of character development. However, the current model also provides an alternative approach to motivational messaging appropriate for physical education teachers and/or coaches who prefer to provide ongoing, or regular, feedback. Put simply, regular character-based reinforcement to athletes is appropriate if positive hope-based messaging is held in tension with temperance-based messaging. The above findings (see Table 1) would suggest that a ratio of approximately 2:1 is ideal.

Conclusion

In sum, the logistic map is a useful model able to mimic complex behaviour. In the present context an adapted form of the logistic map was used to simulate character development under a variety of conditions. Of particular interest were manipulations in: (1) athletes' initial level of character; (2) the threat they perceived; and (3) the motivation they were exposed to. Ultimately, the findings demonstrated the importance of adversity and moderate levels of motivation to induce sustained character development. The present findings also suggested that while significant character development is possible this occurs by many small steps over time. Such findings will be of use to coaches, physical education teachers and sport psychologists alike when seeking to develop training protocols to grow athletes' character. Not only are key elements of such protocols now identified but testable predictions have been made. As such, practitioners now have significant guidance in how to grow athletes' character as a vital element of performance and sporting success more generally.

Conflicts of interest - The authors have no conflicts of interest to declare.

References:

Bhattacharyya, B. K. (2017). *A study on some local and global properties of the chaotic logistic map* [Doctoral dissertation, Gauhati University]. Retrieved from Shodhganga.

Cranmer, G. A., Anzur, C. K., & Sollitto M. (2017). Memorable messages of social support that former high school athletes received from their head coaches. *Communication & Sport, 5*(5), 604–621. https://doi.org/10.1177/2167479516641934.

Curry, L. A., & Snyder, C. R. (2000). Hope takes the field: Mind matters in athletic performances. In C. R. Snyder (Ed.), *Handbook of hope: Theory, measures, and applications* (pp. 243–259). Academic Press.

Curry, L. A., Snyder, C. R., Cook, D. L., Ruby, B. C., & Rehm, M. (1997). The role of hope in student-athlete academic and sport achievement. *Journal of Personality and Social Psychology, 73*, 1257–1267. https://doi.org/10.1037//0022-3514.73.6.1257

d'Elia, F. (2019). The core curriculum of university training to teach physical education in Italy. *Journal of Physical Education and Sport, 19*(Suppl. issue 5), 1755–1758. https://doi.org/10.7752/jpes.2019.s5256

Edwards, T. M., & Jovanovski, A. (2016). Hope as a therapeutic target in counselling–In general and in relation to Christian clients. *International Journal for the Advancement of Counselling, 38*(2), 77–88. https://doi.org/10.1007/s10447-016-9257-8.

Gustafsson, H., Skoog, T., Podlog, L., & Lundqvist, C. (2013). Hope and athlete burnout: Stress and affect as mediators. *Psychology and Sport and Exercise, 14*(5), 640–649. https://doi.org/10.1016/j.psychsport.2013.03.008

Hardman, A., & Jones, C. (2011). *The ethics of sports coaching*. Routledge.

Hardman, A., Jones, C., & Jones, R. (2010). Sports coaching, virtue ethics and emulation. *Physical Education and Sports Pedagogy, 15*(4), 345–359. https://doi.org/10.1080/17408980903535784

Jones, C. (2017). Care and phronesis in teaching and coaching: Dealing with personality disorder. *Sport, Education and Society, 22*(2), 214–229. https://doi.org/10.1080/13573322.2015.1015976

Kassing, J., & Pappas, M. (2007). "Champions are built in the off-season": An exploration of highs school coaches' memorable messages. *Human Communication, 10*(4), 537–546.

Lickona, T. (2001). *What is effective character education?*. The Stony Brook School Symposium on Character. Retrieved July 24, 2021, from https://wicharacter.org/wp-content/uploads/What-is-Effective-Character-Ed-Stonybrook-debate-by-Thomas-Lickona.pdf

Marini, A., Safitri, D., Sujarwo, Zahari, M., Lestari, I., Rihatno, T., Nuraini, S., Iskandar, R., & Ibrahim, N. (2021). Model of character building applied in physical education and sport class: Case in Indonesia. *Journal of Physical Education and Sport, 21*(Suppl. issue 4), 2389–2394. https://doi.org/ 0.7752/jpes.2021.s4320

Mascolo, M. F. (2014). Towards a relational-developmental model of character development. *Human Development, 57*, 347–353. https://doi.org/10.1159/000368989

May, R. M. (1976). Simple mathematical models with very complicated dynamics. *Nature, 261*(5560), 459–467. https://doi.org/10.1038/261459a0

Norwood, M. J. (2001). *The role of hope in exercise adherence, exercise commitment, exercise motivation, and fitness improvement* [Doctoral Thesis, University of Kansas].

Reid, H. L. (2011). *Athletics and philosophy in the ancient world: Contests of virtue*. Routledge.

Richardson, M. J., Dale, R., & Marsh, K. L. (2014). Complex dynamical systems in social and personality psychology: Theory, modelling and analysis. In H. T. Reis & C. M. Judd (Eds.), *Handbook of research methods in social and personality psychology* (2nd edition; pp. 251–280). Cambridge University Press.

Snyder, C. R., Shorey, H. S., Cheavens, J., Pulvers, K. M., Adams, V. H., & Wiklund, C. (2002). Hope and academic success in college. *Journal of Educational Psychology, 94*(4), 820–826. https://doi.org/10.1037/0022-0663.94.4.820

Yeager, J. M., Buxton, J. N., Baltzell, A. L., & Bzdell, W. B. (2001). *Character and coaching: Building virtue in athletic programs.* National Professional Resources Inc.